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6. AUTHOR(S)
Paul R. Garabedian

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
New York University
Courant Institute of Mathematical Sciences
251 Mercer Street
New York, N.Y. 10012

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AFOSR/NM, Building 410
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Dr. Marc Q. Jacobs/NM (202) 767-5025

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13. ABSTRACT *(Maximum 200 words)*

The project was concerned with the development of high performance computer codes for problems in transonic aerodynamics. A practical rule to calculate the wave drag for solutions of the Euler equations was developed from an entropy equality. Symmetric shockless airfoils were analyzed for which uniqueness fails in the transonic case not just for potential flow, but also for the Euler equations.

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COMPUTATIONAL FLUID DYNAMICS and TRANSONIC FLOW

FINAL TECHNICAL REPORT

Period: 1 October 1990 - 30 September 1994

By

Paul R. Garabedian

New York University
Courant Institute of Mathematical Sciences
251 Mercer Street
New York, N.Y. 10012

The addition of artificial viscosity terms to the Euler equations not only ensures numerical convergence, but also models the physical problem correctly. Specifically, the additional terms guarantee that an entropy inequality is satisfied. This can be achieved by upwind differencing in the supersonic region of a transonic flow so that the truncation error represents an artificial viscosity satisfying the entropy inequality. Most Euler solvers, however, employ central differences for all spatial derivatives, and therefore explicit artificial viscosity terms are added to all four equations in order to guarantee convergence. A check on the physical validity of the additional terms is to show that not only is the entropy inequality satisfied for the modified equations, but it also results in a formula for the wave drag depending on the specific form of the artificial viscosity.

Both potential and Euler solvers in our transonic aerodynamics codes make use of the entropy inequality to measure the wave drag. This calculation involves summing a positive definite quantity over the region of flow in which the shock is smeared, avoiding the use of computed flow variables near the trailing edge where there is uncertainty. In more standard approaches the pressure is integrated around the airfoil or over a contour around the shock, but the double integration is more accurate.

Consider the modified Euler equations

$$\rho_t + (\rho u)_x + (\rho v)_y = \nabla \cdot \nu \nabla \rho$$

$$(\rho u)_t + (\rho u^2 + p)_x + (\rho u v)_y = \nabla \cdot \nu \nabla (\rho u)$$

$$(\rho v)_t + (\rho u v)_x + (\rho v^2 + p)_y = \nabla \cdot \nu \nabla (\rho v)$$

$$(\rho E)_t + (\rho u H)_x + (\rho v H)_y = \nabla \cdot \nu \nabla (\rho H) \ ,$$

where artificial viscosity has been added on the right of the form

$$\nabla \cdot \nu \nabla \equiv \frac{\partial}{\partial x}\left(\nu^{(x)} \frac{\partial}{\partial x}\right) + \frac{\partial}{\partial y}\left(\nu^{(y)} \frac{\partial}{\partial y}\right) \ .$$

These equations may be combined to form a fifth equation for the entropy

$$(\rho S)_t + (\rho u S)_x + (\rho v S)_y = \frac{1}{T}\{\nabla \cdot \nu \nabla(\rho H) - (E - TS)\nabla \cdot \nu \nabla \rho$$
$$-u(\nabla \cdot \nu \nabla(\rho u) - u \nabla \cdot \nu \nabla \rho) - v(\nabla \cdot \nu \nabla(\rho v) - v \nabla \cdot \nu \nabla \rho)\} ,$$

whose left-hand side is in conservative form. The right-hand side, however, may be separated into a positive definite term and a divergence term leading to wave drag norms of the form

$$(\rho S)_t + (u\rho S)_x + (v\rho S)_y = \|\nu \nabla U\|^2 + \|\lambda \Delta U\|^2 + \nabla \cdot (\nu S \nabla \rho + \cdots) .$$

An application of the divergence theorem then yields a representation

$$C_D = \int\int (\|\nu \nabla U\|^2 + \|\lambda \Delta U\|^2)\, dx\, dy$$

for the wave drag as an integral of positive terms over any region including the shocks. A corollary of this analysis is that the numerical solution of the Euler equations has to satisfy the entropy inequality asserting that S increases through shocks. The entropy integral for the drag has the advantage in three-dimensional flow that it enables one to distinguish the wave drag from the induced drag, which may be much larger. On a practical mesh it turns out that the entropy formula provides a good estimate of drag in the realistic range of Mach numbers.

Methods for the design of supercritical wing sections have led to lifting airfoils for which two distinct solutions of the Euler equations can be found with visibly different shocks at identical flow conditions. Recently we have applied complex characteristics to construct a symmetric shockless airfoil that has lift at zero angle of attack caused by shocks on the upper and lower surfaces that are not the same. Here again the solution of the Euler equations is not unique. The large size of the supersonic zones swept out by the characteristics is presumably what accounts for the existence of multiple solutions at off-design conditions.

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